R_{II} type recurrence, generalized eigenvalue problem and orthogonal polynomials on the unit circle

M.E.H. Ismail, A. Sri Ranga*

Departamento de Matemática Aplicada, IBILCE, UNESP - Universidade Estadual Paulista, SP, Brazil

Email: `ranga@ibilce.unesp.br`

R_{II} type recurrence relations were introduced by Ismail and Masson. The importance of these recurrence relations were highlighted in a paper by Zhedanov, where he shows that they are connected to generalized eigenvalue problems involving two tri-diagonal matrices. Here, we present a study of a sequence of polynomials $\{P_n\}_{n \geq 0}$ satisfying a special R_{II} type recurrence relation where the zeros of all P_n are simple and lie on the real line. By considering the generalized eigenvalue problem it is shown that associated with any such special R_{II} type recurrence relation there exists a positive measure on the unit circle. The orthogonality property satisfied by the polynomials P_n with respect to this measure is also found. Examples are given to justify the results.